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Top transport in electroweak baryogenesis

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Abstract: In non-supersymmetric models of electroweak baryogenesis the top quark plays a crucial role. Its CP-violating source term can be calculated in the WKB approximation. We investigate the numerical impact of certain discrepancies between computations starting from the Dirac equation and the Schwinger-Keldysh formalism. We also improve on the transport equations, keeping the W-scatterings at finite rate. We apply these results to a model with one Higgs doublet, augmented by dimension-6 operators, and find that the effects discussed here lead to an increase in the baryon asymmetry by a factor of up to about 5.

Keywords: Thermal Field Theory, Baryogenesis, CP violation, Beyond Standard Model.

Contents

1. Introduction

Electroweak baryogenesis [1] is typically described by a set of transport equations, which are fueled by CP-violating source terms. The source terms arise from the CP-violating interactions of particles in the hot plasma with the expanding bubble walls during a first order electroweak phase transition [2]. By diffusion the sources move into the symmetric phase [3], where baryon number violation is fast. For walls much thicker than the inverse transition temperature the wall-plasma interactions can be treated in a WKB approximation, which corresponds to an expansion in gradients of the bubble profile. At first order in gradients a CP-violating shift is induced in the dispersion relations of particles crossing the bubble wall [4]. A (semiclassical) force results, different for particles and antiparticles, which creates a non-zero left-handed quark density in front of the bubble. The weak sphalerons partly transform this left-handed quark density into a baryon asymmetry.

The WKB approach has been widely used to study electroweak baryogenesis in various extensions of the standard model (SM) [5 – 11]. (An alternative approach was followed in ref. [12].) In the simplest manner, the WKB dispersion relations were computed by solving the one-particle Dirac equation to first order in gradients in the CP-violating bubble wall background. In a more rigorous treatment similar dispersion relations were also derived in the Schwinger-Keldysh formalism [13 – 15] (see ref. [16] for some earlier work).

Comparing the dispersion relation of a single Dirac fermion obtained from the Dirac equation (see. e.g. ref. [5]) to that derived from the Schwinger-Keldysh formalism, one observes that the CP-violating part of the latter is somewhat enhanced. In ref. [14] it was shown that this mismatch disappears when the result gained from the Dirac equation is correctly boosted to a general Lorentz frame. Thus in the case of a single Dirac fermion, the full Schwinger-Keldysh result can be obtained in a much simpler way. For realistic models of electroweak baryogenesis, as investigated in the WKB approach, the corrected dispersion relation has only been used in the erratum of ref. [7]. It is therefore important to check to what extent the numerical results of the WKB literature are affected.

In the current paper we study the impact of the modified dispersion relation on the generated baryon asymmetry. To test the numerical significance of this effect we recompute the baryon asymmetry in the SM augmented by dimension-6 operators [10]. Using the correct dispersion relation does enhance the baryon asymmetry by a factor of up to about 2.

We also improve on the transport equations, keeping scatterings with W bosons at a finite rate, which considerably reduces or enhances the baryon asymmetry, depending on the wall velocity. We also show that the position dependence of certain thermal averages in the transport equations has a substantial impact on the baryon asymmetry. Finally, we investigate to what extent the CP-violating source terms are influenced by CP-conserving perturbations in the plasma, an effect that turns out to be negligible. In total, depending on the model parameters, our refinements can increase the baryon asymmetry by a factor of up to about 5.

2. The semiclassical force

Let us review the compution of the dispersion relation in the WKB approach as presented in ref. [14]. We consider a single Dirac fermion, such as the top quark. Its mass changes as it passes the bubble wall. Once the bubble has sufficiently grown, we can approximate the bubble wall by a planar profile. The profile is kink-shaped and characterized by a wall thickness L_w. The problem is most simply treated in the rest frame of the bubble wall. In the presence of CP-violation, the fermion mass term can be complex, i.e. $\mathrm{Re}(\mathcal{M})+i\gamma^5\mathrm{Im}(\mathcal{M})$, where

$$\mathcal{M} = m(z)e^{i\theta(z)} \tag{2.1}$$

and z is the coordinate perpendicular to the bubble wall.

For a particle with momentum much larger than L_w^{-1} we can solve the Dirac equation using a WKB ansatz

$$\Psi \sim e^{-i\omega t + i\int^z p_{cz}(z')dz'} \tag{2.2}$$

and expand in gradients of $\mathcal{M}$. Here p_{cz} is the canonical momentum along the z direction. To simplify the solution we have boosted to the frame where the momentum perpendicular to the wall is zero. Since the typical momentum of a particle in the plasma is on the order of the temperature T, this approach is valid for thick bubbles, i.e. $TL_w \gg 1$. Note that at this stage the fermion is treated as a free particle. Scatterings with particles in the plasma will be incorporated later on by means of the Boltzmann equation.

As shown in ref. [7] the dispersion relation is, to first order in gradients

$$\omega = \sqrt{(p_{cz} - \alpha_{CP})^2 + m^2} \mp \frac{s\theta'}{2}, \tag{2.3}$$

with $\theta' = \partial_z\theta$, $\alpha_{CP} = \alpha' \pm \frac{\theta'}{2}$, and $s = 1$ (-1) for z-spin up (down). The upper (lower) sign corresponds to particles and antiparticles, respectively, which this way get different dispersion relations. The additional phase α is related to an ambiguity in the definition of the canonical momentum, when replacing $\Psi \to e^{i\alpha(z)}\Psi$. It was the main result of refs. [6, 7] that this ambiguity disappears when all quantities are expressed in terms of the kinetic momentum rather than the canonical momentum.

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In ref. [7] the dispersion relation (2.3) was used to compute the semiclassical force, which was then generalized to a Lorentz frame with finite momentum parallel to the wall. In ref. [14] it was pointed out that first eq. (2.3) should be boosted to the general frame and all further manipulations should be carried out later on. This way the dispersion relation of ref. [15] is correctly reproduced.

Since Lorentz invariance is not broken parallel to the wall, we simply have to replace $\omega^2 \to \omega^2 + p_x^2 + p_y^2$. Note that parallel to the wall we do not have to distinguish between kinetic and canonical momentum, i.e. $p_{cx,y} = p_{x,y}$. The dispersion relation (2.3) turns into

$$\omega = \omega_0 \mp s \frac{\theta'}{2} \frac{\omega_{0z}}{\omega_0}, \tag{2.4}$$

where

$$\begin{aligned} \omega_0 &= \sqrt{(p_{cz} - \alpha_{CP})^2 + p_x^2 + p_y^2 + m^2} \\ \omega_{0z} &= \sqrt{(p_{cz} - \alpha_{CP})^2 + m^2}. \end{aligned} \tag{2.5}$$

In the limit $\omega_0 = \omega_{0z}$ we are back at the old result. In the following we show that when written in terms of the kinetic momentum the dependence on α_{CP} still drops.

The physical kinetic z-momentum is given by $p_z = \omega v_{gz}$, where v_{gz}, the group velocity of the WKB wave-packet in the z direction, is given by

$$v_{gz} = \left(\frac{\partial \omega}{\partial p_{cz}}\right)_z = \frac{p_{cz} - \alpha_{CP}}{\omega_0} \left(1 \mp s \frac{\theta'}{2} \frac{\omega_0^2 - \omega_{0z}^2}{\omega_0^2 \omega_{0z}}\right). \tag{2.6}$$

The kinetic momentum then is

$$p_z = (p_{cz} - \alpha) \left(1 \mp s \frac{\theta'}{2\omega_{0z}}\right). \tag{2.7}$$

We can use this expression to replace the canonical momentum in the dispersion relation (2.4). To stress the difference, we introduce a new symbol, E, to denote energy expressed in terms of the kinetic momentum. Defining

$$\begin{aligned} E_0 &= \sqrt{p_z^2 + p_x^2 + p_y^2 + m^2} \\ E_{0z} &= \sqrt{p_z^2 + m^2}, \end{aligned} \tag{2.8}$$

we obtain, to first order in gradients

$$E = E_0 \pm \Delta E = E_0 \mp s \frac{\theta' m^2}{2 E_0 E_{0z}}. \tag{2.9}$$

Notice that the ambiguity related to α_{CP} has disappeared. For the group velocity we now find

$$v_{gz} = \frac{p_z}{E_0} \left(1 \pm s \frac{\theta'}{2} \frac{m^2}{E_0^2 E_{0z}}\right). \tag{2.10}$$

From the canonical equations of motion we can compute the force acting on the particle

$$F_z = \dot{p}_z = \omega \dot{v}_{gz} = \omega \left(\dot{z} \left(\frac{\partial v_{gz}}{\partial z}\right)_{p_{cz}} + \dot{p}_{cz} \left(\frac{\partial v_{gz}}{\partial p_{cz}}\right)_z \right) = \omega \left(v_{gz} \left(\frac{\partial v_{gz}}{\partial z}\right)_{p_{cz}} - \left(\frac{\partial \omega}{\partial z}\right)_{p_{cz}} \left(\frac{\partial v_{gz}}{\partial p_{cz}}\right)_z \right) \tag{2.11}$$

where we have used the fact that ω is constant along the trajectory. Performing the partial derivatives and replacing the canonical by the kinetic momentum, we finally obtain

$$F_z = -\frac{(m^2)'}{2E_0} \pm s\frac{(m^2\theta')'}{2E_0E_{0z}} \mp s\frac{\theta' m^2 (m^2)'}{4E_0^3 E_{0z}}. \tag{2.12}$$

Thus particles and antiparticles experience a different force as they pass the bubble wall. This CP-violating part of the force is second order in derivatives. There is also a CP-conserving part, which is first order in derivatives.

The expressions for the dispersion relation (2.9), the group velocity (2.10), and the semiclassical force (2.12) agree with the results of ref. [15], demonstrating that for a single Dirac fermion the full Schwinger-Keldysh result can be obtained in a simpler way by means of the Dirac equation. This is the main result of this letter.

In the special case $E_0 = E_{0z}$, i.e. when the particle has no momentum parallel to the wall, the full results agree with those of ref. [7]. For a relativistic particle in the plasma E_{0z} contains only roughly a third of the total energy. Keeping correct track of the factors E_{0z} enhances the CP-violating part of the dispersion relation and the force term by a factor of up to about 3. For non-relativistic particles the effect is smaller. This factor has been neglected so far in computations of the baryon asymmetry based on the WKB approximation of the Dirac equation (except for the erratum of ref. [7]). We will demonstrate this enhancement in a numerical example in section 4.

In the next section we discuss the impact of the CP-violating force on the transport equations of particles in the plasma. In a chiral theory, as the SM, interactions are related to the chirality of a particle rather than its spin. Thus it is convenient to label particles in terms of helicity λ, which is close to chirality for relativistic particles. We then have to replace the spin by $s = \lambda \mathrm{sign}(p_z)$ in eqs. (2.9), (2.10) and (2.12).

3. Transport equations

In the derivation of the transport equations we closely follow ref. [7]. A crucial assumption made in that work is that it is the kinetic momentum that is conserved in the scatterings of WKB particles. The equilibrium phase space distributions should therefore also be written in terms of the kinetic momentum. In the wall frame this reads

$$f_i^{(\mathrm{eq})}(\mathbf{x},\mathbf{p}) = \frac{1}{e^{\beta\gamma_w(E_i+v_w p_z)} \pm 1} \tag{3.1}$$

where $\beta = 1/T$ and $\gamma_w = 1/\sqrt{1-v_w^2}$, and plus (minus) refers to fermions (bosons), respectively. We model the perturbations from equilibrium caused by the passage of the bubble wall with a fluid-type ansatz

$$f_i(\mathbf{x},\mathbf{p}) = \frac{1}{e^{\beta[\gamma_w(E_i+v_w p_z)-\mu_i]} \pm 1} + \delta f_i(\mathbf{x},\mathbf{p}). \tag{3.2}$$

The chemical potentials $\mu_i(z)$ describe a local departure from the equilibrium particle density. The perturbations δf_i model a departure from kinetic equilibrium and allow the particles to move in response to the force exerted by the bubble wall. They do not contribute to the particle density, i.e. $\int d^3p\ \delta f_i = 0$. To second order in derivatives, we have to distinguish between particle and antiparticle perturbations, which we can expand as

$$\mu_i = \mu_{i,1e} + \mu_{i,2o} + \mu_{i,2e}, \qquad \delta f_i = \delta f_{i,1e} + \delta f_{i,2o} + \delta f_{i,2e}. \tag{3.3}$$

Notice that the second order perturbations have CP-even and CP-odd parts, which we treat separately.

Let us now concentrate on the Dirac fermion of the last section, so that we can drop the index i to simplify the notation. We expand its distribution function to second order in derivatives as

$$\begin{aligned} f \approx\ & f_{0,v_w} + f'_{0,v_w}(\gamma_w \Delta E - \mu_{1e} - \mu_{2o} - \mu_{2e}) + \frac{1}{2} f''_{0,v_w}(\gamma_w^2 (\Delta E)^2 - 2\gamma_w \Delta E \mu_{1e} + \mu_{1e}^2) \\ & + \delta f_{1e} + \delta f_{2o} + \delta f_{2e}. \end{aligned} \tag{3.4}$$

Here f_{0,v_w} denotes the equilibrium distribution (3.1) where E is replaced by E_0, and $f'_{0,v_w} = (d/dE_0) f_{0,v_w}$. The dependence on the wall velocity is taken exact at this stage.

The evolution of f is governed by the Boltzmann equation

$$\mathbf{L}[f] \equiv (\dot{z}\partial_z + \dot{p}_z \partial_{p_z}) f = \mathbf{C}[f]. \tag{3.5}$$

We look for a stationary solution, so that the explicit time derivative drops. Plugging the ansatz (3.4) into the Boltzmann equation, taking $\dot{z}$ and $\dot{p}_z$ from eqs. (2.10) and (2.12), and subtracting the results of particles and antiparticles, we obtain for the flow part

$$\begin{aligned} \mathbf{L}[f]|_{\mathrm{CP-odd}} =\ & -\frac{p_z}{E_0} f'_{0,v_w} \mu'_2 + \gamma_w v_w \frac{(m^2)'}{2E_0} f''_{0,v_w} \mu_2 + \gamma_w v_w \mathrm{sign}(p_z) \frac{(m^2\theta')'}{2E_0 E_{0z}} f'_{0,v_w} \\ & + \gamma_w v_w \mathrm{sign}(p_z) \frac{\theta' m^2 (m^2)'}{4E_0^2 E_{0z}} \left(\gamma_w f''_{0,v_w} - \frac{f'_{0,v_w}}{E_0} \right) \\ & + \frac{\theta' m^2 |p_z|}{2E_0^2 E_{0z}} \left(\gamma_w f''_{0,v_w} - \frac{f'_{0,v_w}}{E_0} \right) \mu'_1 - \gamma_w v_w \mathrm{sign}(p_z) \frac{(m^2\theta')'}{2E_0 E_{0z}} f''_{0,v_w} \mu_1 \\ & - \gamma_w v_w \mathrm{sign}(p_z) \frac{\theta' m^2 (m^2)'}{4E_0^2 E_{0z}} \left(\gamma_w f'''_{0,v_w} - \frac{f''_{0,v_w}}{E_0} \right) \mu_1 + \frac{p_z}{E_0} \partial_z \delta f_2 - \frac{(m^2)'}{2E_0} \partial_{p_z} \delta f_2 \\ & + \frac{\theta' m^2 |p_z|}{2E_0^3 E_{0z}} \partial_z \delta f_1 + \mathrm{sign}(p_z) \left[\frac{(m^2\theta')'}{2E_0 E_{0z}} - \frac{\theta' m^2 (m^2)'}{4E_0^3 E_{0z}} \right] \partial_{p_z} \delta f_1. \end{aligned} \tag{3.6}$$

Note that the second order perturbations present differences for particles and antiparticles, i.e. $\mu_2 = \mu_{2o} - \bar{\mu}_{2o}$, the same as for δf_2. The CP-even parts drop. For the first order perturbations we take $\mu_1 = \mu_{1e} + \bar{\mu}_{1e}$, etc.

We average the Boltzmann equation over momentum,weighting it by 1 and p_z/E_0. We also expand in the wall velocity, keeping only the linear order, i.e. $f_{0,v_w} \approx f_0 + v_w p_z f'_0$. We then obtain

$$\begin{aligned} v_w K_1 \mu'_2 + v_w K_2 (m^2)' \mu_2 + u'_2 - \langle \mathbf{C}[f] \rangle &= S_\mu \\ -K_4 \mu'_2 + v_w \tilde{K}_5 u'_2 + v_w \tilde{K}_6 (m^2)' u_2 - \left\langle \frac{p_z}{E_0} \mathbf{C}[f] \right\rangle &= S_\theta + S_u \end{aligned} \tag{3.7}$$

with the source terms

$$\begin{aligned} S_\mu &= K_7 \theta' m^2 \mu_1' \\ S_\theta &= -v_w K_8 (m^2 \theta')' + v_w K_9 \theta' m^2 (m^2)' \\ S_u &= -\tilde{K}_{10} m^2 \theta' u_1'. \end{aligned} \tag{3.8}$$

The primes again denote derivatives with respect to z. The sources $S_{\mu,u}$ are related to the first order perturbations. Notice that these are first order in v_w. Formally, $S_{\mu,u}$ are one order higher in gradients than S_θ. It will turn out that they indeed contribute only a small fraction to the total source term. After momentum integration we normalize the resulting equations by the average of the massless Fermi-Dirac distribution

$$\langle X \rangle = \frac{\int d^3p \; X(p)}{\int d^3p f'_{0+}(m=0)}. \tag{3.9}$$

This normalization we also use for bosons to keep the interaction rates for fermions and bosons equal. The plasma velocity we define as

$$u_2 = \left\langle \frac{p_z}{E_0} \delta f_2 \right\rangle. \tag{3.10}$$

The thermal averages read

$$\begin{aligned} K_1 &= -\left\langle \frac{p_z^2}{E_0} f_0'' \right\rangle, & \tilde{K}_6 &= \left[\frac{E_0^2 - p_z^2}{2E_0^3} f_0' \right], \\ K_2 &= \left\langle \frac{f_0''}{2E_0} \right\rangle, & K_7 &= \left\langle \frac{|p_z|}{2E_0^2 E_{0z}} \left(\frac{f_0'}{E_0} - f_0'' \right) \right\rangle, \\ K_3 &= \left\langle \frac{f_0'}{2E_0} \right\rangle, & K_8 &= \left\langle \frac{|p_z| f_0'}{2E_0^2 E_{0z}} \right\rangle, \\ K_4 &= \left\langle \frac{p_z^2}{E_0^2} f_0' \right\rangle, & K_9 &= \left\langle \frac{|p_z|}{4E_0^3 E_{0z}} \left(\frac{f_0'}{E_0} - f_0'' \right) \right\rangle, \\ \tilde{K}_5 &= \left[\frac{p_z^2}{E} f_0' \right], & \tilde{K}_{10} &= \left[\frac{|p_z| f_0}{2E_0^3 E_{0z}} \right]. \end{aligned} \tag{3.11}$$

The averages $\tilde{K}_i$ are related to averages involving δf_2. Since we do not know the momentum dependence of δf_2, we make the additional assumption that these averages factorize and then use eq. (3.10), e.g. $\langle p_z^3 \delta f_2 \rangle \approx [p_z^2 E_0 f_{0,v_w}] u$. We normalize these averages by the massive distribution of the boson or fermion under consideration, i.e. $\int d^3p \; f_{0,v_w}$. Since there is some arbitrariness in this procedure, we will test the impact of these averages, which turns out to be small.[1]

The collision integrals read [7]

$$\begin{aligned} \langle \mathbf{C}[f] \rangle &= \Gamma^{\text{inel}} \sum \mu_{i,2} \\ \left\langle \frac{p_z}{E_0} \mathbf{C}[f] \right\rangle &= -\Gamma^{\text{tot}} u_2, \end{aligned} \tag{3.12}$$

[1]Depending on how we precisely treat the averages involving δf_2, there can also arise a source term of the form $(m^2)'\theta' u_1$. We do not discuss it in more detail since the source terms related to the first order perturbations are small anyway.

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where $\Gamma^{\rm inel}$ and $\Gamma^{\rm tot}$ are the inelastic and total interaction rates, respectively. The negative sign in front of $\Gamma^{\rm tot}$ is related to our sign convention for the plasma velocity (3.10).

The transport equations of the first order perturbations look very similar to eq. (3.7)

$$\begin{aligned} v_w K_1 \mu_1' + v_w K_2 (m^2)' \mu_1 + u_1' - \Gamma^{\rm inel} \sum \mu_{i,1} &= v_w K_3 (m^2)' \\ -K_4 \mu_1' + v_w \tilde{K}_5 u_1' + v_w \tilde{K}_6 (m^2)' u_1 + \Gamma^{\rm tot} u_1 &= 0. \end{aligned} \tag{3.13}$$

The source term is now first order in derivatives and CP-even. Note that here also the quite large annihilation rates enter in $\Gamma^{\rm inel}$.

In eqs. (3.7) and (3.13) we can approximately eliminate the plasma velocity to obtain diffusion equations for chemical potentials. From the coefficient of the μ'' term we can read off the diffusion constant as [7]

$$D = \frac{K_4}{K_1 \Gamma^{\rm tot}}. \tag{3.14}$$

Our source terms (3.8) agree with those obtained from the Schwinger-Keldysh formalism. However, in ref. [15] there is one extra source term, related to the gradient renormalization of the Wigner function. This term seems to be missing in the Dirac equation approach. It is of order m^4, like the K_9-part of S_θ. We will demonstrate in the next section that these terms are subleading.

4. Top transport: an example

We now apply the general results (3.7) and (3.13) to top transport in an effective SM with dimension-6 operators [17–20, 10]. The model contains a single Higgs doublet, whose potential is stabilized by a ϕ^6 interaction

$$V(\phi) = -\frac{\mu^2}{2}\phi^2 + \frac{\lambda}{4}\phi^4 + \frac{1}{8M^2}\phi^6. \tag{4.1}$$

This potential has two free parameters, the suppression scale M of the dimension-6 operator and the quartic coupling λ. The latter can be eliminated in terms of the physical Higgs mass m_H. Since the potential is stabilized by the ϕ^6 term, λ can be negative. In this case a barrier in Higgs potential is present at tree-level, which triggers a first order electroweak phase transition. Computing the 1-loop thermal potential, it was shown in ref. [10] that the phase transition is strong enough to avoid baryon number washout, i.e. $\xi = \langle\phi\rangle_{T_c}/T_c > 1.1$ [21], if $M \lesssim 850$ GeV and $m_H = 115\,$GeV. Taking $M = 500\,$GeV, a strong phase transition is present for $m_H \lesssim 180\,$GeV. Thus the model allows for a strong phase transition in a large part of its parameter space. In ref. [10] also the wall thickness has been determined, showing that $3 \lesssim L_w T_c \lesssim 16$. The gradient expansion discussed in section 2 is therefore justified in almost the full parameter space. The thinnest walls correspond to a very strong phase transition, $\xi \sim 3$, where the model is close to metastability of the symmetric phase. In the following we will approximate the wall profile by a hyperbolic tangent, $\phi(z) = (v_c/2)(1 - \tanh(z/L_w))$.

Dimension-6 operators also induce new sources of CP-violation. In addition to the ordinary Yukawa interaction of the top quark, $y_t \Phi t^c q_3$, we have an operator

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$(x_t/M^2)(\Phi^\dagger\Phi)\Phi t^c q_3$ [18]. We denote the relative phase between the two couplings as $\varphi_t = \arg(yx^*)$. Then the top develops a position dependent complex phase θ_t along the bubble wall $\phi(z)$, with

$$\tan\theta_t(z) \approx \sin\varphi_t \frac{\phi^2(z)}{2M^2}\left|\frac{x_t}{y_t}\right|. \tag{4.2}$$

So all necessary ingredients are present to apply the formalism discussed in the previous sections.

For the generation of the baryon asymmetry, the most important particle species are the left- and right-handed top quarks, and the Higgs bosons. We will show that the latter have only a minor impact. We ignore leptons, which are only produced by small Yukawa couplings. In contrast to all previous investigations we include the W scatterings with a finite rate Γ_W. This procedure allows us to study the perturbations of bottom and top quarks separately. The top quark source is no longer locked to the bottom degrees of freedom, which would lead to a larger or smaller baryon asymmetry, depending on the wall velocity. The other interactions we take into account are the top Yukawa interaction, Γ_y, the weak and strong sphalerons, Γ_{ws} and Γ_{ss}, the top helicity flips, Γ_m, and Higgs number violation Γ_h. The latter two are only present in the broken phase.

In a first step we compute the left-handed quark density, assuming that baryon number is conserved. Later on, the left-handed quark density will be converted into a baryon asymmetry by the weak sphalerons. The transport equations for chemical potentials of left-handed SU(2) doublet tops $\mu_{t,2}$, left-handed SU(2) doublet bottoms $\mu_{b,2}$, left-handed SU(2) singlet tops $\mu_{t^c,2}$, Higgs bosons $\mu_{h,2}$, and the corresponding plasma velocities read

$$\begin{aligned}
3v_wK_{1,t}\mu'_{t,2} + 3v_wK_{2,t}(m_t^2)'\mu_{t,2} + 3u'_{t,2}&\\
-3\Gamma_y(\mu_{t,2}+\mu_{t^c,2}+\mu_{h,2}) - 6\Gamma_m(\mu_{t,2}+\mu_{t^c,2}) - 3\Gamma_W(\mu_{t,2}-\mu_{b,2})&\\
-3\Gamma_{ss}[(1+9K_{1,t})\mu_{t,2}+(1+9K_{1,b})\mu_{b,2}+(1-9K_{1,t})\mu_{t^c,2}] &= 3K_{7,t}\theta'_t m_t^2\mu'_{t,1}\\
3v_wK_{1,b}\mu'_{b,2} + 3u'_{b,2}&\\
-3\Gamma_y(\mu_{b,2}+\mu_{t^c,2}+\mu_{h,2}) - 3\Gamma_W(\mu_{b,2}-\mu_{t,2})&\\
-3\Gamma_{ss}[(1+9K_{1,t})\mu_{t,2}+(1+9K_{1,b})\mu_{b,2}+(1-9K_{1,t})\mu_{t^c,2}] &= 0\\
3v_wK_{1,t}\mu'_{t^c,2} + 3v_wK_{2,t}(m_t^2)'\mu_{t^c,2} + 3u'_{t^c,2}&\\
-3\Gamma_y(\mu_{t,2}+\mu_{b,2}+2\mu_{t^c,2}+2\mu_{h,2}) - 6\Gamma_m(\mu_{t,2}+\mu_{t^c,2})&\\
-3\Gamma_{ss}[(1+9K_{1,t})\mu_{t,2}+(1+9K_{1,b})\mu_{b,2}+(1-9K_{1,t})\mu_{t^c,2}] &= 3K_{7,t}\theta'_t m_t^2\mu'_{t^c,1}\\
2v_wK_{1,h}\mu'_{h,2} + 2u'_{h,2}&\\
-3\Gamma_y(\mu_{t,2}+\mu_{b,2}+2\mu_{t^c,2}+2\mu_{h,2}) - 2\Gamma_h\mu_{h,2} &= 0
\end{aligned}\tag{4.3}$$

$$\begin{aligned}
-3K_{4,t}\mu'_{t,2} + 3v_w\tilde{K}_{5,t}u'_{t,2} + 3v_w\tilde{K}_{6,t}(m_t^2)'u_{t,2} + 3\Gamma_t^{\text{tot}}u_{t,2} &=\\
= -3v_wK_{8,t}(m_t^2\theta'_t)' + 3v_wK_{9,t}\theta'_t m_t^2(m_t^2)' - 3\tilde{K}_{10,t}m_t^2\theta'_t u'_{1,t}&\\
-3K_{4,b}\mu'_{b,2} + 3v_w\tilde{K}_{5,b}u'_{b,2} + 3\Gamma_b^{\text{tot}}u_{b,2} &= 0\\
-3K_{4,t}\mu'_{t^c,2} + 3v_w\tilde{K}_{5,t}u'_{t^c,2} + 3v_w\tilde{K}_{6,t}(m_t^2)'u_{t^c,2} + 3\Gamma_t^{\text{tot}}u_{t^c,2} &=\\
= -3v_wK_{8,t}(m_t^2\theta'_t)' + 3v_wK_{9,t}\theta'_t m_t^2(m_t^2)' - 3\tilde{K}_{10,t}m_t^2\theta'_t u'_{1,t^c}&\\
-2K_{4,h}\mu'_{h,2} + 2v_w\tilde{K}_{5,h}u'_{h,2} + 2\Gamma_h^{\text{tot}}u_{h,2} &= 0
\end{aligned}\tag{4.4}$$

In eqs. (4.4) Γ_W can be neglected since the plasma velocities of t and b are damped by the much faster gluon scatterings.[2] We have used baryon number conservation to express the sphaleron interaction in terms of $\mu_{t,2}$, $\mu_{b,2}$ and $\mu_{t^c,2}$ [22]. A possible source term for the bottom quark is suppressed by (m_b/m_t) and therefore neglected.

The first order perturbations of t can be computed from

$$\begin{aligned} 3v_w K_{1,t}\mu'_{t,1} + 3v_w K_{2,t}(m_t^2)'\mu_{t,1} + 3u'_{t,1} - 3\Gamma_t^{\rm tot}\mu_{t,1} &= 3v_w K_{3,t}(m_t^2)' \\ -3K_{4,t}\mu'_{t,1} + 3v_w \tilde{K}_{5,t}u'_{t,1} + 3v_w\tilde{K}_{6,t}(m_t^2)'u_{t,1} + 3\Gamma_t^{\rm tot}u_{t,1} &= 0. \end{aligned} \tag{4.5}$$

The damping of $\mu_{t,1}$ is dominated by gluon annihilation, the rate of which we have approximated by $\Gamma_t^{\rm tot}$. Other scatterings have been neglected. To this approximation the chemical potentials of t and t^c are identical. This guarantees that no direct source for baryon number is induced. Such a source can be generated if $\mu_{t,1} \neq \mu_{t^c,1}$. It leads to spurious effects in the baryon asymmetry. Its appearance shows that an inconsistent approximation pattern has been used.

We can now compute the chemical potential of left-handed quarks, $\mu_{B_L} = \mu_{q_1,2} + \mu_{q_2,2} + (\mu_{t,2} + \mu_{b,2})/2$. Assuming again baryon number conservation, we obtain

$$\mu_{B_L} = \frac{1}{2}(1+4K_{1,t})\mu_{t,2} + \frac{1}{2}(1+4K_{1,b})\mu_{b,2} - 2K_{1,t}\mu_{t^c,2}. \tag{4.6}$$

The baryon asymmetry is then given by [7]

$$\eta_B = \frac{n_B}{s} = \frac{405\Gamma_{ws}}{4\pi^2 v_w g_* T}\int_0^\infty dz\ \mu_{B_L}(z)e^{-\nu z}, \tag{4.7}$$

where is Γ_{ws} the weak sphaleron rate and $\nu = 45\Gamma_{ws}/(4v_w)$. The effective number of degrees of freedom in the plasma is $g_* = 106.75$. In eq. (4.7) the weak sphaleron rate has been suddenly switched off in the broken phase, $z < 0$. The exponential factor in the integrand accounts for the relaxation of the baryon number if the wall moves very slowly. Note that we have performed our computation in the wall frame. Therefore, strictly speaking eq. (4.7) gives the baryon asymmetry in that frame. To first order in v_w this is identical to the baryon asymmetry in the plasma frame.

In our numerical evaluations we use the following values for the weak sphaleron rate [23], the strong sphaleron rate [24], the top Yukawa rate [22], the top helicity flip rate, the Higgs number violating rate [22], the quark diffusion constant [4] and the Higgs diffusion constant [7]

$$\begin{aligned} &\Gamma_{ws} = 1.0\times 10^{-6}T, && \Gamma_{ss} = 4.9\times 10^{-4}T, \\ &\Gamma_y = 4.2\times 10^{-3}T, && \Gamma_m = \frac{m_t^2(z,T)}{63T}, \\ &\Gamma_h = \frac{m_W^2(z,T)}{50T}, && D_q = \frac{6}{T}, \\ &D_h = \frac{20}{T}. \end{aligned} \tag{4.8}$$

[2] In the numerical evaluations we have included a term $3\Gamma_W(u_{t,2} - u_{b,2})$ which affects results only at the few percent level.

We use eq. (3.14) to infer the total interaction rates from the diffusion constants. In this procedure we evaluate the thermal averages at $z = 0$, i.e. in the center of the bubble wall. The W scatterings we approximate as $\Gamma_W = \Gamma_h^{\rm tot}$. The bottom quark is taken as massless, and the Higgses we count as 2 massless complex degrees of freedom. The rates of eq. (4.8) have been computed in the plasma frame. We assume that, to leading order in v_w, they can also be used in the wall frame.

To demonstrate the relevance of the various contributions to the full transport equations, we compare the baryon asymmetry computed in different approximations for two typical parameter settings. We take $|x_t| = 1$ and maximal CP violation $\sin\varphi_t = 1$. Figure 1 shows η_B as a function of the wall velocity v_w. The other parameters we have chosen as $\xi = 1.5$, $M = 6$ and $L_w = 8$. These values correspond to a setting where the baryon asymmetry is close to the observed value $\eta_B = (8.9 \pm 0.4) \times 10^{-11}$ [25, 26]. In figure 2 we use $\xi = 2.5$, $M = 6$ and $L_w = 3$, i.e. a very strong phase transition with a small wall width. In both figures the bold solid line (a) indicates η_B using the source terms S_θ and keeping the full z-dependence of the thermal averages (3.11).

In (b) we drop the space-dependence of the thermal averages. We rather evaluate them at the center of the bubble wall, i.e. $K_{i,t}(z) \equiv K_{i,t}(z = 0)$. Formally, the space-dependence of the thermal averages is a higher order effect in gradients. But this approximation considerably underestimates the baryon asymmetry, especially for small wall velocities and thin bubble walls. The full z-dependence reduces the impact of the wall velocity on η_B.

The long-dashed line (c) shows the result when we resubstitute $E_{0z} \to E_0$, going back to the dispersion relation of ref. [7]. This would considerably reduce the baryon asymmetry, in particular for weaker phase transitions (figure 1).

Neglecting the Higgs bosons in the transport eqs. (4.3) and (4.4) leads to a reduction of η_B by $\simeq 10\%$ (d), almost independent of the wall velocity and the strength of the phase transition.

Taking the W scatterings to equilibrium (e) has a substantial effect on the resulting baryon asymmetry, especially for strong phase transitions. In figure 2 it overestimates η_B by a factor of almost 2 for $v_w < 0.1$. For large wall velocities there is an underestimate of η_B by a similar size. Keeping W scatterings finite results in a much milder v_w-dependence of the baryon asymmetry.

The dash-dotted line (f) adds the contributions of $S_\mu + S_u$ to line (a). The effect of these source terms is quite small, consistent with the fact that they are of higher order in gradients. They enhance the baryon asymmetry in the whole v_w-range only by a few percent.

Line (g) shows the effect of switching off the terms proportional to $\tilde{K}_5$ and $\tilde{K}_6$. If these terms are neglected, the final result is reduced by a contribution proportional to the wall velocity. It demonstrates that the precise treatment of the averages involving δf has only a minor impact on the baryon asymmetry.[3]

[3]Numerically there is also not much difference to the prescription used, for instance, in ref. [10], where plasma velocities were included in the fluid ansatz, rather than using a general δf. Then, for example, the u_2' term in eq. (3.7) obtains an additional coefficient ~ 1.1.

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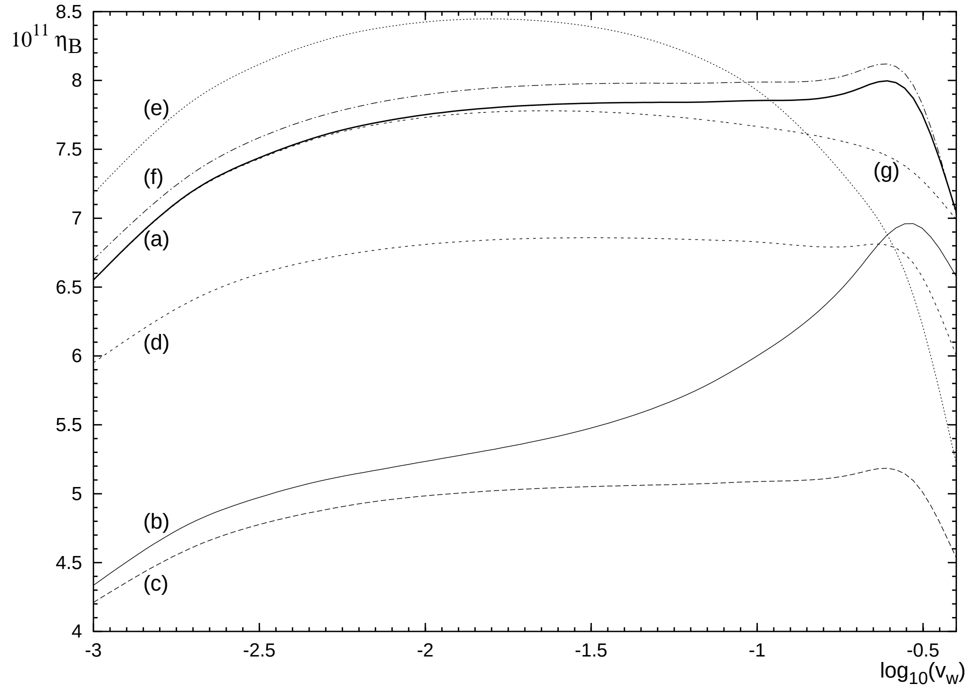

Figure 1: The baryon asymmetry as a function of v_w for $\xi = 1.5$, $M = 6$ and $L_{\rm w} = 8$. The labeling is explained in the text.

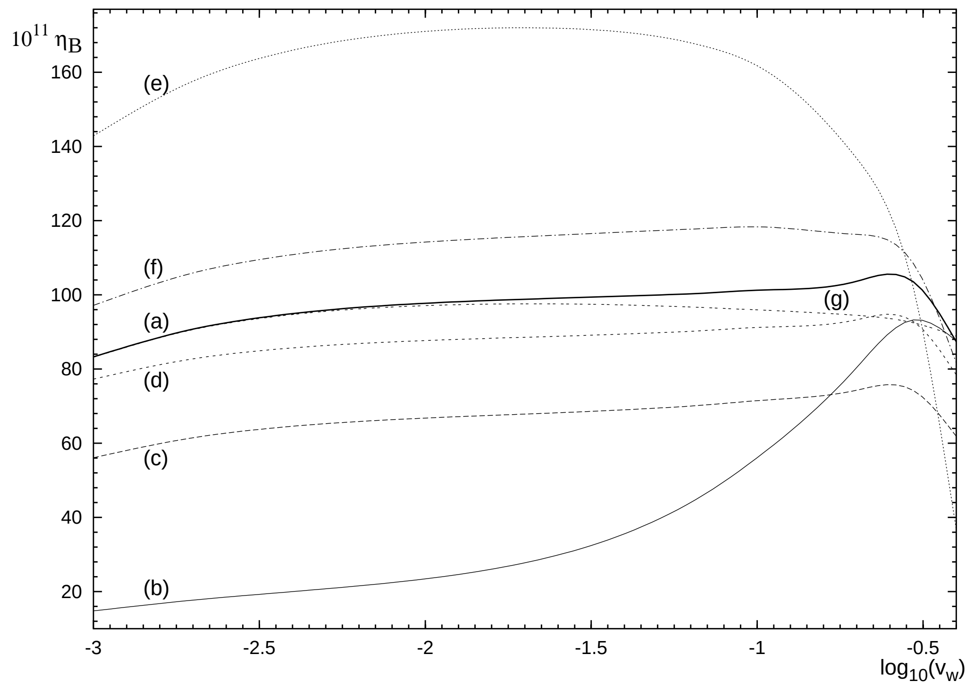

Figure 2: The baryon asymmetry as a function of v_w for $\xi = 2.5$, $M = 6$ and $L_{\rm w} = 3$.

Altogether the examples demonstrate that the leading contribution to η_B comes from the source S_θ. The baryon asymmetry gets considerably enhanced by using the dispersions relation with the correct factors of E_{0z} and keeping the space-dependence of the thermal averages. The finite W scattering rate has a sizable effect, the direction of which depends on the wall velocity. The resulting v_w-dependence of the baryon asymmetry is rather mild. The baryon asymmetry grows slowly with increasing v_{w} and reaches a maximum at $v_w \simeq$ 0.2–0.3. Taking the Higgs bosons or the $S_\mu + S_u$ sources into account is less important. Their effect is not larger than typical uncertainties from higher order terms in the gradient expansion.

The source S_θ consists of two parts, proportional to K_8 and K_9. The latter has an additional factor m^2, leading to an extra suppression, in particular for weak phase transitions. For instance, taking $v_w = 0.1$ and the parameter set of figure 1, the K_9-part contributes only about 15% to the total baryon asymmetry. As indicated earlier, there is an extra source term in ref. [15], which is related to the gradient renormalization of the Wigner function. It also has an extra factor of m^2 and therefore should also be sub-leading in our case.

Of course, the baryon asymmetry also depends on the precise values of the interaction rates (4.8). For instance, reducing the quark diffusion constant by 10% leads to an about 7% reduction in the baryon asymmetry (taking $v_w = 0.1$ and the parameter set of figure 1). Changing Γ_W by 10% affects η_B to less than 1%, even for $v_w \sim 0.01$, where the impact of the W scatterings is particularly large.

Figure 3 displays the baryon asymmetry in the SM with a low cut-off as a function of the cut-off scale M. We consider two different Higgs masses $m_H = 115$ GeV and $m_H = 150$ GeV and two wall velocities $v_w = 0.01$ and 0.3. For each value of M the corresponding strength of the phase transition and bubble width are computed as in ref. [10]. As expected η_B increases rapidly with decreasing cut-off scale M. The asymmetry has only a minor dependence on the wall velocity. In both cases it is possible to generate the measured baryon asymmetry for a reasonably small value of M.

5. Conclusions

We have improved on the computation of the baryon asymmetry arising from top transport. Making use of the one-particle Dirac equation in the wall background, we have reviewed the calculation of CP-violating source term in the WKB approximation. When the top dispersion relation is correctly boosted to a general Lorentz frame, the Schwinger-Keldysh result [13, 15] for the semiclassical force term is obtained in eq. (2.12) [14]. The CP-violating source term is enhanced with respect to ref. [6]. We have only considered the case of a single Dirac fermion, but our results should simply generalize to mixing fermions, such as to the charginos in the MSSM.

In the WKB approach one cannot obtain the extra source term of ref. [15], which is related to the gradient renormalization of the Wigner function. In the case of top transport this term is subleading since it is of order m^4. In this approach, of course, one also cannot obtain source terms related to quantum mechanical oscillations between different fermion

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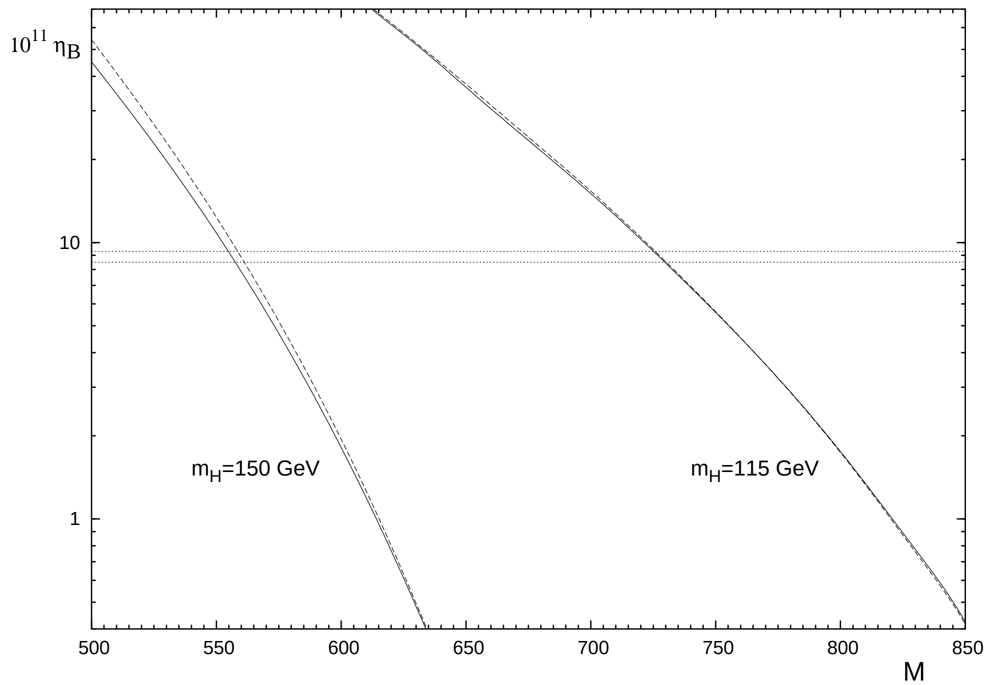

Figure 3: The baryon asymmetry in the SM with low cut-off for two different Higgs masses as a function of M (in units of GeV) for $v_w = 0.01$ (solid) and $v_w = 0.3$ (dashed). The horizontal lines indicate the error band of the observed value.

flavors. In the case of the top quark this effect is obviously not present, but it can be relevant for the charginos in the MSSM [27].

We have demonstrated the numerical significance of the corrected dispersion relations in the SM augmented by dimension-6 operators. This effect alone enhances the baryon asymmetry by a factor of up to about 2. We have also improved on the transport equations, keeping scatterings with W bosons at a finite rate. Depending on the wall velocity and the wall thickness, putting the W scatterings to equilibrium (as was done so far in the literature) can increase or decrease the baryon asymmetry by a factor of 2. It would be interesting to study the impact of this effect in supersymmetric models, where the SU(2) supergauge interactions have been put to equilibrium as well.

We have shown that the position dependence of the thermal averages in the transport equations has a substantial impact on the baryon asymmetry, even though it is formally a higher order effect in the gradient expansion. Finally, the influence of the Higgs bosons on transport turned out to be small, as is the contribution of the sources $S_{\mu,u}$ (3.8). In total, depending on the model parameters, our refinements can increase the baryon asymmetry by a factor of up to about 5.

The rather large impact of the precise treatment of the W scattering rate and the space-dependence of the thermal averages probably indicate that there is still a substantial uncertainty related to transport.

In a forthcoming publication we will apply the framework presented here to compute

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the baryon asymmetry in the two Higgs doublet model [28].

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